Crisis control: Preventing chaos-induced capsizing of a ship

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(Received 11 July 1994)

Responses of many man-made systems, such as ships or oil-drilling platforms, when subject to irregularly time varying environments, can be described by irregularly driven dynamical systems. Consequently, failures of such systems (e.g., capsize of a ship or collapse of a platform), under increasingly severe environmental conditions, come about when the system state escapes from a destroyed chaotic attractor located in some favorable region of the phase space. In this paper we propose a control strategy, based on a previous method of chaos control, which can prevent such failures from taking place. The key feature of our strategy is the incorporation of prediction of the evolution of the environment. This makes possible effective operation of the control even when the temporal behavior of the environment has substantial irregularity. We illustrate the ideas using ship capsizing as an example.

PACS number(s): 05.45.+b

Imagine a ship rolling in lateral ocean waves. Its dynamics can be modeled by the following nonlinear oscillator,

$$\ddot{x}+\nu\dot{x}+\omega^2(x-\alpha x^3)=W(t)\ , \tag{1}$$

where x is a variable characterizing the angle from the ship mast to the vertical direction, ν is the friction coefficient, ω is the eigenfrequency of small vibrations around the origin, α denotes the strength of nonlinearity, and $W(t)$ represents the effect of the ocean waves impinging on the ship. In particular, we shall be interested in the case where the ocean waves have substantial temporal irregularity and drift superposed on their mean periodic time dependence.

In the absence of waves, i.e., $W(t)=0$, for a small displacement in x, the subsequent motion damps out and the ship reverts back to its upright position. For large displacements in x, gravity overcomes buoyancy and x tends to the attractor at $|x|=\infty$. When this occurs we say the ship has capsized. Models of similar type have been used in other studies of ship dynamics [1–3].

We assume that the irregular ocean waves have the following form:

$$W(t)=f(t)[1+\epsilon_a g(t)]\sin\phi(t)\equiv F(t)\sin\phi(t)\ , \tag{2}$$

where $F(t)$ is the amplitude, with $f(t)$ it slowly varying component and $g(t)$ a fast irregularly varying component; the phase $\phi(t)$ is determined by

$$\dot{\phi}(t)=\Omega+\epsilon_p h(t)\ . \tag{3}$$

Here again $h(t)$ is an irregular function of time. In what follows, we (somewhat arbitrarily) model the temporally irregular functions $g(t)$ and $h(t)$ as outputs of well-known chaotic systems. Specifically, we use

$$g(t)=\frac{y(t)-\langle y(t)\rangle}{\sqrt{\langle[y(t)-\langle y(t)\rangle]^2\rangle}}\ ,$$

where $y(t)$ is a solution of the following driven Duffing system:

$$\ddot{y}+0.05\dot{y}+y^3=7.5\cos t\ ,$$

and

$$h(t)=\frac{z_1(t)-\langle z_1(t)\rangle}{\sqrt{\langle[z_1(t)-\langle z_1(t)\rangle]^2\rangle}}\ ,$$

with $z_1(t)$ taken from the Rössler system,

$$\dot{z}_1=-z_2-z_3\ ,$$
$$\dot{z}_2=z_1+0.398z_2\ ,$$
$$\dot{z}_3=2+z_3(z_1-4)\ .$$

The symbol $\langle\ \rangle$ denotes temporal average. Since $g(t)$ and $h(t)$ are normalized, ϵ_a and ϵ_p measure the relative strength of amplitude irregularity and that of phase irregularity, respectively. (We emphasize that our use of low-dimensional chaotic systems to produce the irregularly varying $g(t)$ and $h(t)$ is merely a convenience, and there should be no essential difference in what follows if these functions are stochastic or are produced by some high-dimensional chaotic processes.)

For $\epsilon_a=\epsilon_p=0$ and $f(t)=f_0$, where f_0 is a constant, we have purely sinusoidal waves, and the dynamics in this case will be used as the basis for control later when irregularities are introduced into the system. The evolution of Eq. (1) with increasing wave amplitude f_0 and $\epsilon_a=\epsilon_p=0$ is shown in the bifurcation diagram in Fig. 1. (The values of the system parameters used in the numeri-

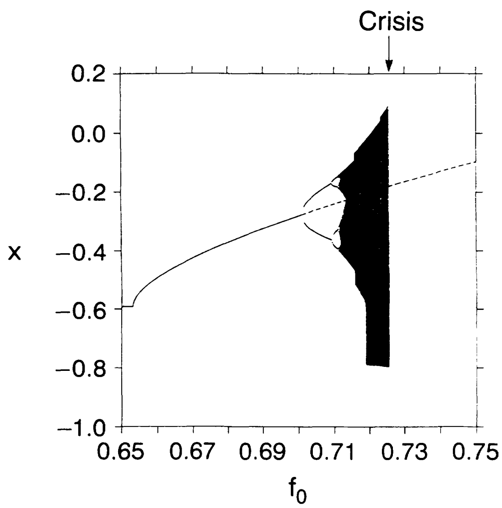

FIG. 1. Bifurcation diagram of Eq. (1). The dashed line indicates the unstable period one orbit after the period doubling bifurcation.

cal computations are quoted in the caption of Fig. 3.) The surface of section here is taken every time $W(t)$ crosses zero with $dW(t)/dt>0$ (i.e., $\Omega t_n = 2n\pi$). For $0<f_0<0.7$ the oscillation of the ship is periodic with the period equal to that of the wave. We henceforth call this orbit the period one orbit. Following a period doubling cascade the ship response becomes chaotic. At $f_0 \approx 0.726$ a crisis takes place in which the bounded chaotic attractor is destroyed by colliding with its basin boundary. As the wave amplitude increases past this crisis value, since no bounded attractor exists in the system, almost all initial conditions tend to the attractor at $|x|=\infty$ (i.e., the ship capsizes). Also shown in Fig. 1 as a dashed curve is the period one orbit after it loses stability. Now suppose that $f(t)$ is a gradually increasing function of time starting from $f(t)=0$. We anticipate that in the absence of control the ship will capsize some time after $f(t)$ exceeds the crisis value. To prevent this, one starts to apply control to stabilize the period one orbit in Fig. 1 when the wave is still small and continues to do so as the wave increases. As a result, we show that the ship survives waves whose amplitude significantly exceeds the crisis value. More importantly, by incorporating a prediction feature to be detailed below, and suitably modifying the control procedure of [4,5], we are able to prevent capsizing in the presence of substantial wave irregularities.

The equation of motion for the variable x, when control is applied, is again Eq. (1), but with the right-hand side replaced by $W(t)+C(t)$, where $C(t)$ is the control. We take $C(t)$ to be a constant between successive crossings of the surface of section. In practical terms, $C(t)$ can be thought of as the balancing force provided by temporally shifting ballast on the ship, and the value of $C(t)$ is assumed to be bounded between $-C_0$ and C_0, i.e., $-C_0 \le C(t) \le C_0$. In fact, we are interested in the case where C_0 is small compared to the force exerted by the waves. We take the surface of section to specify the system state at times $t=t_n$ such that $W(t)=0$ and $dW(t)/dt>0$. Assuming that $f(t)>0$ and that the right-hand side of Eq. (3) is positive this corresponds to $\phi(t_n)=2n\pi$ [see Eq. (2)]. Figure 2 illustrates the prediction and control method. A fixed point of the Poincaré map and its stable and unstable directions are constructed for $\epsilon_a=\epsilon_p=0$ and $f(t)=f(t_n)$. Here we imagine that $f(t)$ can be treated as a constant in the interval $t_n<t<t_{n+1}$. (Note that due to the slow variation of $f(t)$ the fixed point location at $t=t_n$ changes with n [6].) By letting $\epsilon_a \neq 0$ and $\epsilon_p \neq 0$, we introduce irregularity into the wave. The task now is to stabilize the motion around the fixed point. Suppose that at $t=t_n$, the system state is denoted by $\mathbf{z}_n$, where $\mathbf{z}=(x,y=\dot{x})$. From observation of waves propagating toward the ship, we assume that we can make an accurate prediction of $W(t)$ for $t_n \le t \le t_{n+1}$. Integrating Eq. (1) with this predicted $W(t)$, and with various values of $C(t)=\hat{C}$, where $\hat{C}$ is a constant in the interval $[-C_0, C_0]$, we obtain images of the system state on the next surface of section at $t=t_{n+1}$, which form a curve as shown in Fig. 2. The value $\hat{C}=C_n$ of the control that we actually apply to the ship is chosen so that the system state falls on the stable direction of the fixed point.

Figure 3(a) shows part of the wave for $\epsilon_a=0.15$, $\epsilon_p=0.1$, and $f(t)$ a linearly increasing function of t. As shown in Fig. 3(b), in the absence of control, the ship capsizes in several cycles of the ocean waves. Using control with prediction the capsizing is prevented in Fig. 3(b). A more extended controlled orbit is shown in Fig. 3(c). Figure 3(d) shows $C(t)$ as a function of t. Comparing with Fig. 3(a) we see that $C(t)$ is much smaller than the wave amplitude $W(t)$.

In the example above we have tacitly assumed that the wave $W(t)$ can be observed and predicted with certainty in the short term (i.e., from t_n to t_{n+1}). In practice, a more likely situation is that the prediction is imperfect. We now attempt to assess the effect of imperfect prediction. For specificity, we consider only amplitude irregularity in Eq. (2), which we assume to be

$$g(t)=\frac{(1-\gamma)g_o(t)+\gamma g_u(t)}{\sqrt{(1-\gamma)^2+\gamma^2}}, \tag{4}$$

where $g_o(t)$ and $g_u(t)$ are two different normalized trajectory realizations (originating from two different initial

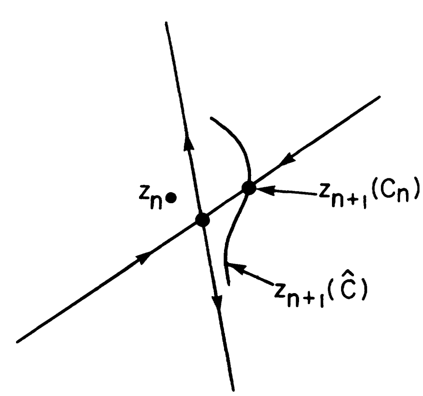

FIG. 2. Schematic of the control method.

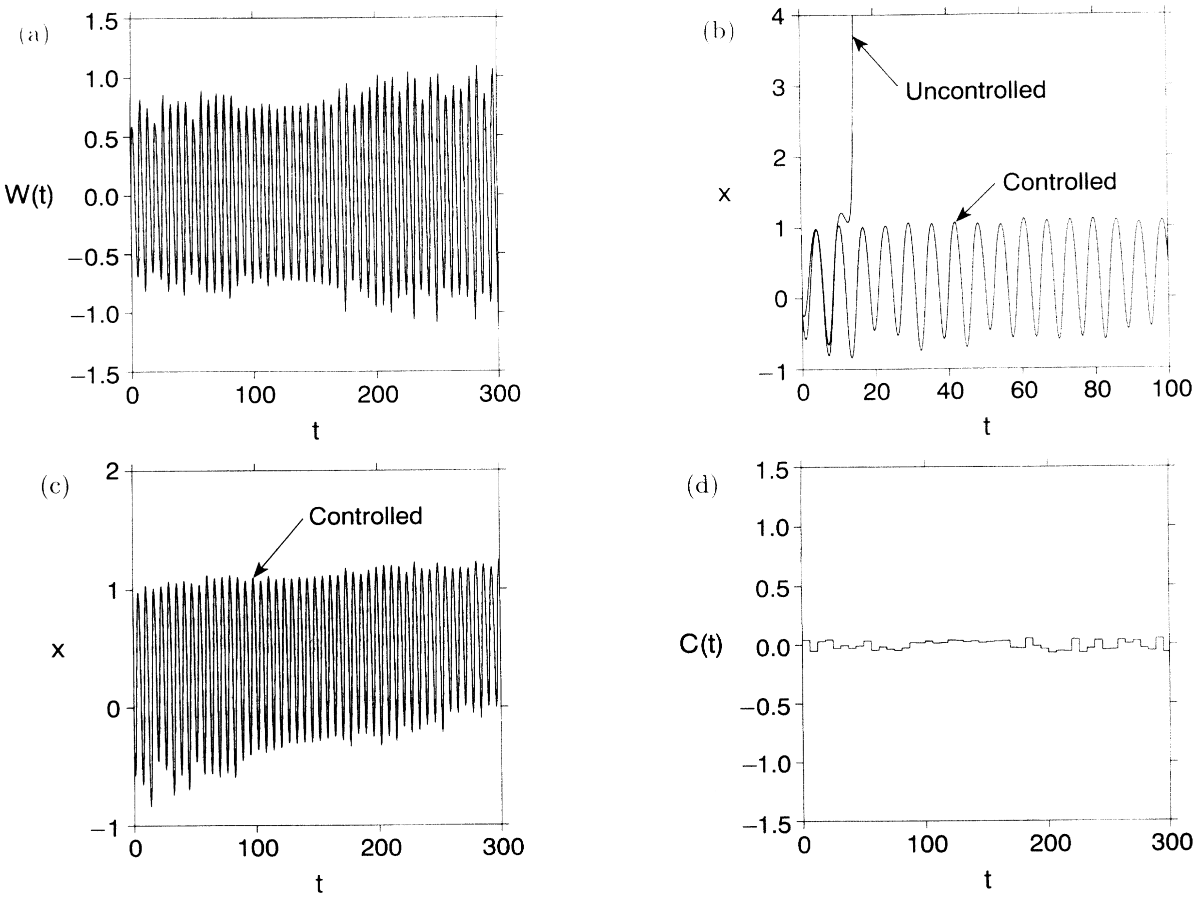

FIG. 3(a) Segment of the irregular increasing wave with $\epsilon_a=0.15$, $\epsilon_p=0.1$, and $f(t)$ a linearly increasing function of t, (b) controlled together with uncontrolled orbit, (c) more extended plot of controlled orbit, and (d) $C(t)$ versus t. The following parameter values are chosen for numerical computations: $\nu=0.5$, $\omega=\Omega=1.0$, $\alpha=1.0$, and integration step size $=2\pi/200$.

conditions) from the same Duffing chaotic attractor, with $g_o(t)$ being the "observed" and $g_u(t)$ the "unobserved" components of $g(t)$, and γ measures the relative strength of these two components. The imperfect prediction of $W(t)$ is assumed to be given by replacing the true $g(t)$ defined in Eq. (4) by $g_o(t)$. To design a control we integrate the equation of motion assuming $g(t)=g_o(t)$. The actual trajectory however is obtained by including $g_u(t)$ in the integration after $C(t)$ has been chosen. Let $\langle T_{\text{survival}}\rangle$ denote the average survival time in wave cycles calculated for ten different realizations of $g_o(t)$ and $g_u(t)$. Our results for $f(t)=f_0=0.9$ and $\epsilon_a=0.1$ are as follows. For $\gamma\leq 0.06$ we have $\langle T_{\text{survival}}\rangle\geq 400$. That is, the control method is robust with respect to less than or equal to 6% of uncertainty. The effectiveness of control deteriorates rapidly as γ increases past 0.06. In particular, we find that $\langle T_{\text{survival}}\rangle\approx 100$ for $\gamma=0.07$, $\langle T_{\text{survival}}\rangle\approx 30$ for $\gamma=0.08$, $\langle T_{\text{survival}}\rangle\approx 20$ for $\gamma=0.1$, $\langle T_{\text{survival}}\rangle\approx 10$ for $\gamma=0.3$, and $\langle T_{\text{survival}}\rangle\approx 2$ for $\gamma=0.0$.

To conclude we remark that, although ship capsizing has been used here to illustrate the ideas of the control strategy, other situations are also candidates for the approach in this paper. In particular, our method is potentially important in applications where a momentary loss of control due to environmental irregularity can result in a catastrophic failure of the system.

This work was supported by the Department of Energy (Office of Scientific Computing, Office of Energy Research), and by the Office of Naval Research.

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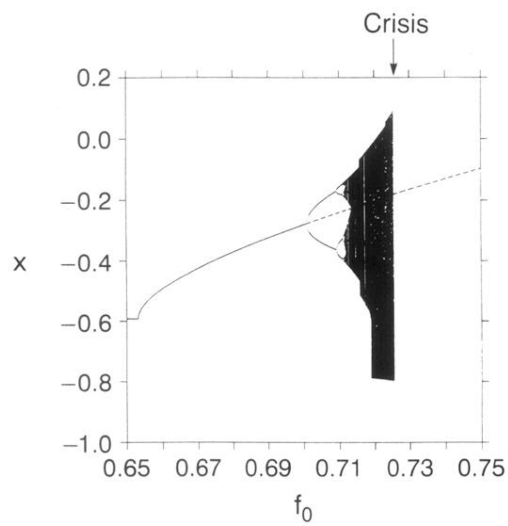

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